

Direct and Partial Variations

Mathematics 10

by
Ray Mah

DIRECT VARIATION SALESPERSON

Val

$$\begin{array}{r} \text{Hourly Rate} \\ \times \text{Hours Worked} \\ \hline = \text{Total Income} \end{array}$$

PARTIAL VARIATION SALESPERSON

$$\begin{array}{r} \text{Basic Income} \\ + \text{Percentage of Sales} \\ \hline = \text{Total Income} \end{array}$$

In order to help you understand the content of this unit, Val and Sal have kindly volunteered to assist us by providing a simple but direct comparative illustration.

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TEACHING MATERIALS
from the
STEWART RESOURCES CENTRE



This unit is taught upon completion of the unit on linear functions. Direct and partial variations are applications of linear functions.

Foundational Objective:

To use the knowledge of linear functions and equations to solve problems involving direct and partial variation.

Learning Objectives:

Curriculum Guide (p. 24)

19. To identify, describe, and interpret examples of direct variation in real world situations.
20. To solve proportions involving direct variation.
21. To solve problems involving direct variation.
22. To identify partial problems involving partial variation.
23. To solve problems involving partial variation.

See Curriculum Guide, p. 124-127 for instructional notes, examples and ideas for adaptive dimensions.

References:

Holtmath 10 c1987,

Addison-Wesley Mathematics 10, c1987

Math Matters 10, Nelson Canada, c1990 (Multi-text approach is recommended)

This is a Yes: Concept Attainment. Sheryl Mills. Saskatoon: Saskatchewan Professional Development Unit and Saskatchewan Instructional Development and Research Unit, 1991.

Instructional Strategies:

(See Curriculum Guide, p. 248-250). In addition to lecturing, practice, and drill, the following instructional strategies are very suitable:

- cooperative learning
- concept attainment (a planning form is attached)
- problem solving
- compare and contrast

Assessment and Evaluation:

(See Curriculum Guide, p. 9-16 and p. 73-92)

It is recommended that a variety of methods is used. An excellent collection of templates for assessment and evaluation is included in the curriculum guide.

Objective 19:

To identify, describe, and interpret examples of direct variation in real world situations.

Lesson Plan:

1. Review of prerequisite skills
2. Develop working definitions of new terms
3. Work out some examples
4. Practice

1. Review of prerequisite skills:

- Introduce the lesson with a quick review of the definitions of relations, functions, mapping, ordered pairs, graph of a function, etc.

2. Definitions:

- In every relation there is a variation between the two variables. When the value of one variable changes, there is a corresponding change in the value of the other variable. Demonstrate with familiar direct variation such as:
 - distance travelled varies directly as time.
 - weekly wage varies directly as hours worked.
 - volume of a gas varies directly as its Kelvin temperature.
 - hotel cost varies directly as the number of days.
 - the expansion of a metal rod varies directly as the change in temperature.
 - the circumference of a circle varies directly as the radius
- List some relations that are not direct variations such as:
 - the speed and time on a trip of 100 km
 - the weight bearing capacity of beam and its length
 - the force of attraction between two bodies and the distance of separation

(NOTE: Concept attainment works well here)

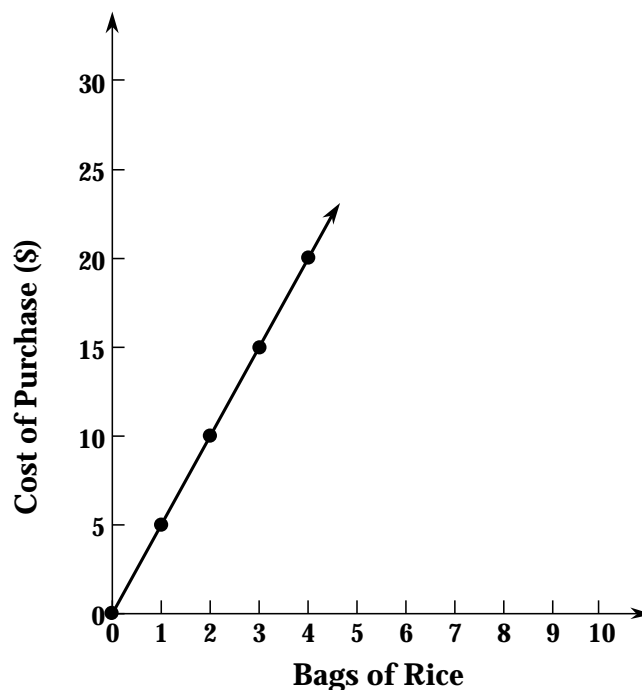
3. Worked example:

Wild rice retails at \$5.00 per 500g bag. Construct a table of values relating cost and number of bags purchased. Graph the variation. Is it a direct variation? What is the constant of variation?

Table of Values

Bags	0	1	2	3	4
Cost \$	0	5	10	15	20

Graph of the Variation



NOTE: Since the graph is a straight line starting at the origin, it is a direct variation. The constant of variation is 5. The equation of the line is $y = 5x$

Summary: The type of variation can be identified by the graph of the relation. The graph of a direct variation is a straight line and starts at the origin. The slope of the line is the constant of variation. The equation of a direct variation is of the form $y = mx$, more commonly written as $y = kx$.

4. Practice:

- To enforce the fact that the graph of a direct variation is linear and starts at the origin, provide the students with an assortment of different graphs to identify. (See page 179, Holtmath 10).
- In addition to the practice problems in math texts, direct variation problems can be found in physics and chemistry texts.

Objective 20:

To solve proportions involving direct variation.

Lesson Plan:

1. Review of prerequisite skills
2. Develop working definitions of new terms and symbols
3. Worked examples
4. Practice

1. Review of prerequisite skills

- Review the definition of a ratio, ways representing ratios.

2. Develop working definitions of new terms

- Present two sets similar to the ones below to the class. Ask the students to provide more ordered pairs for each set. Define the terms means and extremes. Ask the class to compute the product of the means and that of the extremes for each pair. Get the class to list the attributes of Set A.

Set A	Set B
(1/2, 2/4), (3/4, 75/100), (5:3, 20:12), (\$8/h, \$320/40h) (dime/dozen, dollar/120)	(1/2, 1/3), (1/10, 9/100) (5:3, 12:20), (100km/h, 180km/2h)

- An equality of ratios, such as $x/y = w/z$ is called a proportion.
- The proportionality may be written as $x:y = w:z$.
- Since $x/y = w/z$ then there is a number k such that $x/y = k$ and $w/z = k$.
- The constant K is called the constant of proportionality.

3. Worked Example 1:

The following table is a direct variation. Determine the constant of proportionality.

x	y	y/x
1	5	1/5
2	10	2/10 = 1/5
3	15	3/15 = 1/5
4	20	4/20 = 1/5

The constant of proportionality 1/5.

Partially Worked Example 2:

- The key to successful problem solving is the ability to identify the relating variables. Provide the students with plenty of opportunity to practice. When we use a gas barbecue, propane burns with oxygen to produce heat, carbon dioxide and water vapour. For complete combustion of propane, one unit of propane reacts with 5 units of oxygen to produce 3 units of carbon dioxide and 4 units of water vapour.
 - a. How much oxygen is needed to burn 10 kg of propane?
 - b. How much carbon dioxide is produced?

Solution:

- a. The amount of oxygen needed varies directly as the amount of propane present.
Let G = propane gas and P - propane gas.
 $G \propto P$ or $G = kP$

The oxygen to propane ratio is 5 to 1. We can set the proportion as 5:1 = G:10kg
or
form the equation $5/1 = G/10\text{kg}$. Solving for G, we find that 50 kg of oxygen is needed.
(Ask the students to provide alternate methods, eg, table of values, graphically)

- b. Assign part b, perhaps as group work.

4. Practice

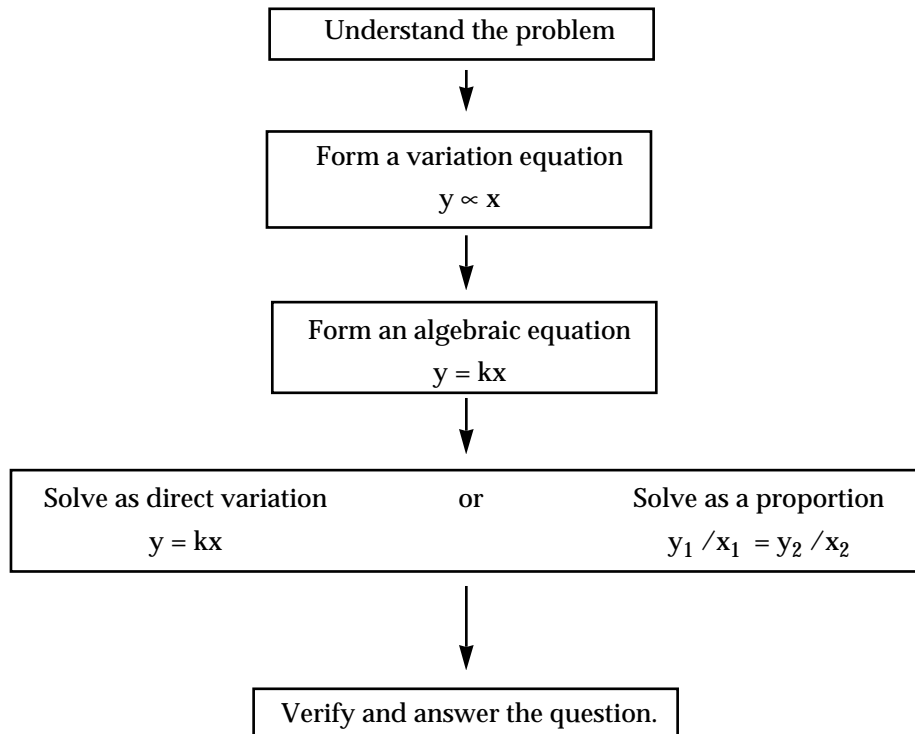
- ... Holtmath 10, p. 180-181
- ... A. W. Math 10, p. 226-227
- ... Math Matters 10, p. 277-279

Chemistry problems involving masses and moles of molecules are excellent for practice.

Objective 21:

To solve proportions involving direct variation.

Systematic problem solving techniques should be emphasized. It is important to identify the independent and dependent variables. Once the relating variables are identified the problem may be solved as direct variation or as a proportion. Some students may want to follow a flow chart such as the one below:



Practice questions are readily available in math and science texts. Ask each student to concoct a number of problems for the class to solve. Make sure that they are direct variations.

Objective 22:

To identify partial problems involving partial variation.

Lesson Plan:

1. Review of prerequisite skills
2. Develop working definitions of new terms
3. Work out some examples
4. Practice

1. Review of prerequisite skills

- Mastery of direct variation is essential.

2. Develop working definitions of new terms

- Divide the chalkboard into halves with a vertical line.
- Sketch a number of graphs, placing graphs of partial variations on one side and the rest on the other side.
- Student will quickly distinguish the two sets.
- Establish the fact:
 - The graph of partial variation is a straight line and the starting point is not the origin. The equation is of the form $y = mx + b$ where b is not zero.

A question to ask:

In a direct variation, what is the effect on the second variable if the first is doubled?

Is the effect the same in a partial variation?

Examples of partial variations:

- car rental charge = basic daily rate plus number of kilometres driven.
- salesman's income = basic monthly income plus percent of sales.
- speed of sound in air = 331m/s plus 0.59m/s for each degree Celsius above zero.
- banquet cost = fixed hall rental plus number of plates of food served.

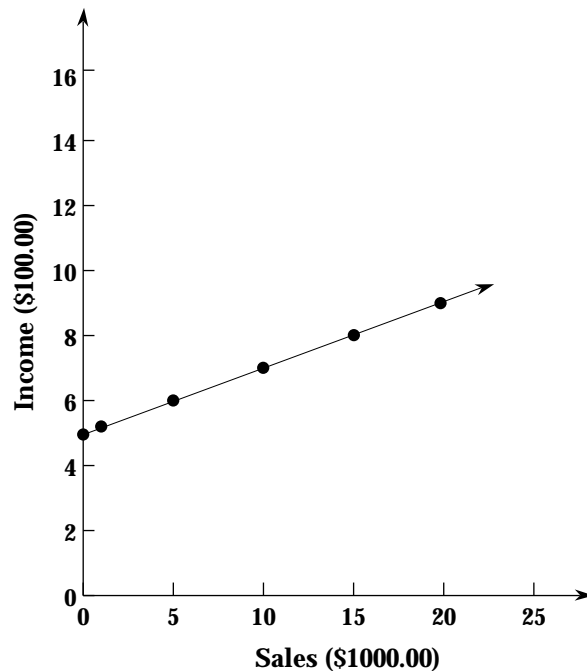
These are not partial variations:

- a teacher's monthly salary
- fuel consumption and speed
- the air pressure and altitude
- the volume of a gas and temperature
- the stopping distance of a car and the speed of the car

3. Worked example:

- A car salesman earns \$500.00/month plus 2% of total sales. Construct a table of values, graph the variation.

Sales	0	1000	5000	10 000	15 000	20 000
Commission	0	20	100	200	300	400
Basic income	500	500	500	500	500	500
Total income	500	520	600	700	800	900



Extension:

- Compare the above with an alternate earning formula which offers no basic monthly income but increases the commission rate from 2% to 3% of the sales.

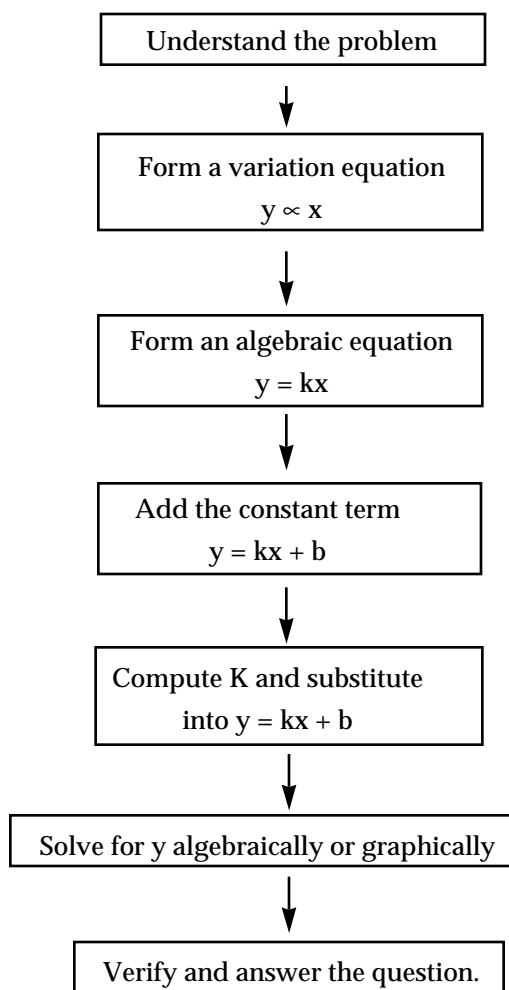
4. Practice

- ... Holtmath, p. 183
- ... Math Matter 10, p. 281-283
- ... A. W. Math 10, p. 230-231 - Addison Wesley
- ... Questions from physics texts

Objective 23:

To solve problems involving partial variation.

- Treat partial variations as linear equations in slope-intercept form, $y = mx + b$. Students need to be able to identify the independent variable, dependent variable, the constant term (y - intercept) and to calculate the constant of proportion (slope). Using a graphing calculator, the problem can easily be solved.
- Convince the students to follow a problem solving strategy. One approach is to follow a flow chart.



(SUGGESTION: Post the flow chart on the wall for a few days.)

Worked example:

Hockey jerseys are priced at \$50.00 each plus \$1.50 per letter. How much will Jerry Desjarlais' jersey cost if his last name is sewn on the back?

Understand the problem:

Given: \$50.00/jersey and \$1.50/letter

Find: Cost of jersey with DESJARLAIS on it.

Form a variation equation

Cost varies directly as number of letters

$$C \propto L$$

Form an algebraic equation

$$C = kL$$

Add the constant term

$$C = kL + 50$$

Compute k and substitute

k = cost per letter

$$= \$1.50$$

$$C = 1.50L + 50$$

Solve for y algebraically or graphically

Algebraically

$$C = 1.50L + 50, L = \text{number of letters in DESJARLAIS} = 10$$

$$\text{By substitution we obtain } C = 1.50 \cdot 10 + 50 = 15 + 50 = 60$$

Verify and answer the question

$$\text{Cost of letters} = 10 \cdot 1.50 = 15.00$$

$$\text{Cost of jersey} = \underline{50.00}$$

$$\text{Total cost} = 65.00$$

Practice questions

- ... Holtmath, p. 183
- ... Math Matter 10, p. 281-282
- ... A. W. Math 10, p. 230-231
- ... Questions from physics texts

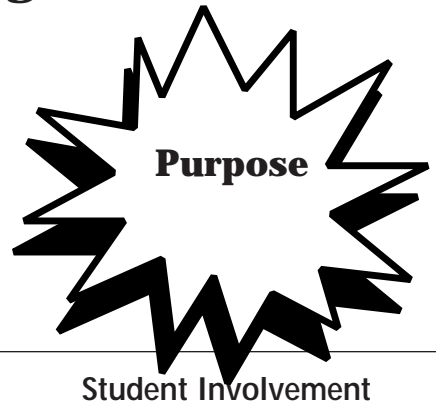
Concept Attainment Lesson Planning Worksheet

Subject

Unit or Theme

Concept Name

Critical Attributes



Student Involvement

Non-Critical Attributes



DATA SET

Examples	Non-Examples

Concrete

GRAPHIC with labels

ABSTRACT

COMBINATION

Presentation

Debriefing

Follow-up

Evaluation
Process
Content
Lesson

Notes

Refinement

Introduction

To meet a need for resources for the new Math 10 curriculum, the Saskatchewan Teachers' Federation, in cooperation with Saskatchewan Education, Training and Employment, initiated the development of teacher-prepared unit plans.

A group of teachers who had piloted the course in 1992-93 were invited to a two and a half day workshop in August, 1993 at the STF. The teachers worked alone or in pairs to develop a plan for a section of the course.

Jim Beamer, University of Saskatchewan, and Lyle Markowski, Saskatchewan Education, Training and Employment, acted as resource persons for the workshop.