Arc's Midpoint Turns Kinetic, Adds Applied to Theoretic…

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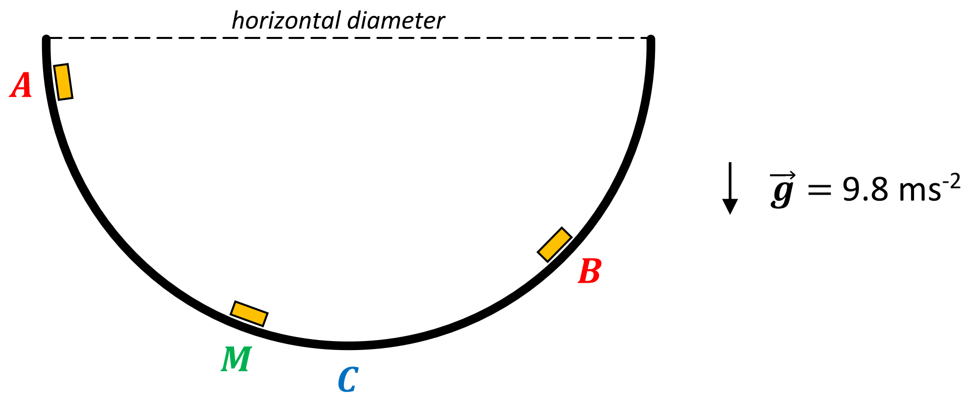
An object, initially at rest, slides from the top of a vertical semicircular path with a horizontal diameter (see diagram). At points ***A***, ***B***, ***C***, *and* ***M*** along the semicircle, the object has kinetic energy in joules $a = 62\ \mathrm{J}$, $b = 638\ \mathrm{J}$, $c = 962\ \mathrm{J}$, and $\mu\ \mathrm{J}$, respectively. If ***C*** is the lowest point of the path, and ***M*** is equidistant from ***A*** and ***B***, determine the **exact value** of μ. Assume that the path is frictionless and the air resistance is negligible.

Diagram



The arc midpoint computation[1] approach to solving this problem finds

$$2\mu = \sqrt{(962\ \mathrm{J} + 638\mathrm{J})(962\ \mathrm{J} + 62\ \mathrm{J})} + \sqrt{(962\ \mathrm{J} - 638\ \mathrm{J})(962\ \mathrm{J} - 62\ \mathrm{J})}\text{, and } \mu = 910\ \mathrm{J}$$

1. Verify the answer using alternative approach. **Compare** solutions.

2. Show that the values of **kinetic energy** a, b, c, and μ satisfy

$$2\mu = \sqrt{(c+a)(c+b)} \pm \sqrt{(c-a)(c-b)} \qquad (1)$$

3. Specify when (1) requires the sum of radicals, and when it requires their difference.

[1] http://mathcentral.uregina.ca/RR/database/RR.09.10/akulov2.html